TURBULENCE SINGULARITIES AT MODERATE REYNOLDS NUMBERS

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Degeneration and spectral dynamics in the rotating coordinate system of a large-scale vortex are investigated.

The interaction between large-scale and fine-scale vortices reduces to adiabatic [1] for very large Reynolds numbers. As is known, after elimination of the adiabatic interaction from the dynamic equations in the "direct interaction" approximation, the Komogorov–Obukhov law is successfully obtained for spectra in the wave number inertial range. However, for moderate values of the Reynolds number the interaction between the large and small-scale vortices can be of different nature in principle.

A large (permanent [2]) vortex produces a Coriolis force field in the domain of space that it occupies, which is due to the rotational motion of the vortex. The comparatively slow motion of a large vortex affords time for the process of radiation of inertial waves to be realized by small-scale fluctuations in the Coriolis force field of this vortex [3, 4]. Such inertial waves transfer the energy being emitted by their fluctuations to the viscous layers. These can be the "viscous superlayers" (free viscous layers that are the interfacial boundaries between zones with intense and weak vorticity (see [5])) as well as viscous layers on solid walls. It is known that the energy brought by the inertial waves to the viscous layers dissipates effectively therein; the papers [3-6] are devoted to an examination of this process, say. It will later be shown that the wave mechanism of energy redistribution is capable of suppressing the cascade mechanism during a finite time.

It turns out here that suppression will occur only upon satisfaction of the condition $Re \lesssim 4\cdot 10^3$. We later call the Reynolds number satisfying this condition moderate. If the cascade mechanism is suppressed mainly by the wave mechanism, then in place of a power-law dependence in the law of fluctuation homogeneous turbulence energy attenuation an exponential type attenuation law is realized. Then, when the inertial waves extract the energy principally in the viscous layers on fixed walls (such that the mean distances between these layers can be considered invariant), the index of exponential attenuation is independent of Re.

1. SUPPRESSION OF CASCADE ENERGY TRANSFER BY WAVE ENERGY

If there is a stable (metastable) rotation in a given space domain that can be characterized by a mean angular velocity Ω, then the velocity (and pressure) field fluctuations will generate inertial waves in the Coriolis force field [3-6]. Wave generation in an incompressible fluid is associated here with the fact that the rotation tends to suppress hydrodynamic gradients along its direction and, therefore, the appearance of these gradients (inevitable in turbulent motion) involves the appearance of forces tending to their annihilation and the cancellation of these gradients also results in the disappearance of the forces they caused [7].

Part of the energy entrained by the inertial waves dissipates in the viscous layers and part, being reflected, returns to the volume occupied by the fluid [3-6]. Attenuation of the turbulent fluctuation kinetic energy as a result of its removal by inertial waves can be described by the equation [3]

$$d\overline{u^2}/dt = -a\,\frac{(\nu\Omega)^{1/2}}{L}\,\overline{u^2}, \tag{1}$$

where ν is the viscosity, L is the mean distance between viscous layers, and a is a dimensionless constant. If Ω and L are considered constants, then there follows from (1)

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$$\overline{u^2} \sim \exp - \frac{a(\nu\Omega)^{1/2}}{L} t. \quad (2)$$

This attenuation can be taken into account in the equation for the spectral tensor in the "external friction" approximation [3, 8]

$$\partial E_{ij}(k, t)/\partial t = -2\nu k^2 E_{ij} + T_{ij}(k, t) - \lambda E_{ij}, \quad (3)$$

where

$$\lambda = a(\nu\Omega)^{1/2}/L;$$

and T_{ij} characterizes the spectral energy transfer due to cascade effects. The examination is conducted here in a reference system connected to the "rotating" large vortex in which the spectral dynamics being investigated further is formed. The "external friction" reflects the interaction between inertial effects and viscous layers and is a phenomenological addition to the spectral equation [3, 8]. Since the inertial term in the Navier-Stokes equations is quadratic in the velocity, then for small E_{ij} the functional T_{ij} is a homogeneous functional of order 3/2 of E_{ij}. Let us make the substitution

$$E'_{ij}(k, t) = E_{ij}(k, t) \exp - \lambda t. \quad (4)$$

Substituting (4) into (3) and neglecting the direct action of viscosity, i.e., the term $-2\nu k^2 E_{ij}$, we obtain

$$\partial E'_{ij}(k, t)/\partial t = T_{ij}(k, t) \exp - \lambda t/2. \quad (5)$$

Now, we make the replacement of the time

$$t' = \frac{2}{\lambda}(1 - \exp - \lambda t/2). \quad (6)$$

Then it follows from (5) that

$$\partial E'_{ij}/\partial t = T_{ij}(k, t'). \quad (7)$$

Therefore, the replacements (4) and (6) reduce the problem with "external friction" to an analogous problem without friction (the initial conditions are evidently identical). As $t \to \infty$ there follows $t' \to 2/\lambda$ from (6) such that the whole evolution described by the equation with "external friction" (3) is stacked in the interval $(0, 2/\lambda)$ of the evolution described by (7) (i.e., the equation without "external friction"). It is shown in [9] in a model example that there is a certain characteristic time of cascade process development:

$$t_c \approx 5.2 \cdot 2\pi/\Omega.$$

Taking the preceding into account, an estimate can be made of the condition under which the cascade process does not develop successfully (because of the action of the "external friction"). This condition has the form

$$2/\lambda \lesssim 5.2 \cdot 2\pi/\Omega. \quad (8)$$

Substituting its value for λ, we obtain

$$2L/a(\nu\Omega)^{1/2} \lesssim 2\pi/\Omega. \quad (9)$$

Introducing the Reynolds number

$$\mathrm{Re} = \left(\frac{L}{2}\right)(L\Omega/2)/\nu,$$

we rewrite condition (9) in the form

$$\mathrm{Re} \lesssim 66a^2.$$

The approximate value for the dimensionless constant a can be taken from the experimental data described in [3] where attenuation of turbulence was investigated behind a cascade

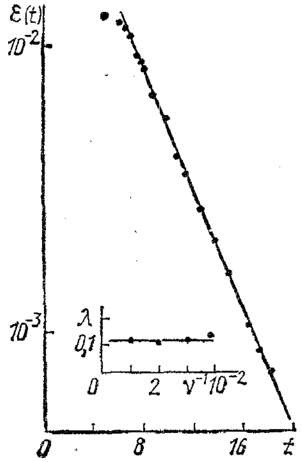

Fig. 1. Turbulence attenuation generated by Taylor—Green vortex.

in a rotating vessel. In this experiment a ≈ 8 is obtained. Therefore the condition of cascade transfer suppression by wave has the form

$$\mathrm{Re} \lesssim 4 \cdot 10^3. \tag{10}$$

2. TURBULENCE ATTENUATION BY PERMANENT VORTICES

The mean parameters for the largest vortices (whose dimensions are comparable to the dimensions of the motion domain) vary slowly with time as compared with changes in $\bar{u}^2$ [2]. These vortices acquire a metastable state in attenuating turbulence because of the action of inertial forces. These forces (we provisionally denote them by f) are equilibrated by viscous friction on the walls (boundaries) of the motion domain. Hence, for metastable (permanent) vortices the equation

$$\nu \Delta v_p = f \tag{11}$$

can be written, where v_p is the velocity field of the permanent vortices. We do not know the explicit form. However, it is clear that f has inertial origination and therefore depends weakly on the viscosity (ν). Consequently, approximately

$$\Omega_p \sim \nu^{-1}, \tag{12}$$

where Ω_p is the effective angular velocity of the fluid rotation in the permanent vortices.

One of the simplest numerical models of homogeneous turbulence is the Taylor—Green vortex [9]. A result obtained by using this model (Sec. 1) has already been utilized above. A flow of the form

$$u_1(x, 0) = \cos x_1 \sin x_2 \cos x_3, \; u_2(x, 0) = -\sin x_1 \cos x_2 \cos x_3, \; u_3(x, 0) = 0$$

in all space is selected therein as the initial condition to the Navier—Stokes equations. The periodicity property is invariant and can be constrained to an examination of the motion in a cube with edge (2π) under periodic boundary conditions. For t > 0 the velocity field becomes three-dimensional, where the vortex lines are extended. Results of a numerical computation of such a problem by using a spectral method are presented in [9]. For the dimensionless time t > 5 the rate of fluctuation energy dissipation $\varepsilon = d\bar{u}^2/dt$ decreases monotonically (the growth of ε for t < 5 is related to buildup of the cascade). If it is assumed that turbulence attenuation is determined by the wave mechanism described in the preceding section, then the attenuation law ε will also be exponential with the same exponent as for $\bar{u}^2$. Data taken from [9] but processed with (2) taken into account (a semilogarithmic scale is selected for this) are presented in the figure. It is seen that the attenuation law is actually approximated well by an exponential. Data for ν^{-1} (dimensionless) = 100 are presented in the figure. There are also data for ν^{-1} = 100, 200, 300, 400 in [7]. Attenuation for them is also approximated well by an exponential, where the attenuation index λ is practically independent of ν^{-1} (see the figure). The independence of λ from ν^{-1} is in conformity with (12) since we obtain by the substitution of the value of Ω_p in place of Ω in the representation for $\lambda = a(\nu\Omega)^{1/2}/L$ and taking account of (12) that λ should be independent of ν. It is known that the large vortices introduce the main contribution to the energy. In what way is dissipation from fine-scale fluctuations capable of substantial influence on dissipation as a whole under conditions of a suppressed cascade process? The fact is that under such conditions the rotational transfer of fine-scale pulsations by a large vortex velocity field is evidently not simply an adiabatic [1] but a governing factor in the mechanism of energy removal by inertial waves since their generation is impossible without it. And, therefore, the energetic relation (energy transmission) between large- and fine-scale vortices is not interrupted but changes its nature: it becomes inertial-wave from cascade.

It should be noted that in full-scale experiments the data on turbulence attenuation behind gratings are approximated quite satisfactorily by power-law dependences for $Re < 10^4$ (see [10, 11], say). The question of how much turbulence generated by the Taylor–Green vortex [9] differs from grating turbulence is not clear. Data of a grating experiment in [12] together with a power-law approximation allow a good approximation by the exponential dependence also. The exponent in this dependence is practically independent of Re (compare with the above elucidation) although the experiments were performed for $Re < 2 \cdot 10^3$. Perhaps the applicability or nonapplicability of the reasoning expressed here to grating turbulence depends on channel geometry (i.e., on the ratio of the grating step to the transverse channel dimension)?

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